XVIII.

ON THE GROUP OF REAL LINEAR TRANSFORMATIONS WHOSE INVARIANT IS AN ALTERNATE BILINEAR FORM.

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Let G denote the group of linear automorphic transformations of the alternate bilinear form

$$\mathfrak{J} = \sum_{1}^{2n}{}_r \sum_{1}^{2n}{}_s (c_{rs} - c_{sr})\, x_r\, y_s$$

with cogrediant variables and of non-zero determinant. On page 575 *et seq.*, Volume XLVI. of the *Mathematische Annalen,* I have shown that a transformation of group G can be generated by the repetition of an infinitesimal transformation of group G if, and only if, it is the second power of a transformation of group G. I now find, if $\mathfrak{J}$ is real, that the same theorem holds for the sub-group of real transformations of group G. That is, if $\mathfrak{J}$ is real, a real transformation of group G can be generated by the repetition of a real infinitesimal transformation of group G if, and only if, it is the second power of a real transformation of this group. Furthermore, if $\mathfrak{J}$ is real, the second power of a real transformation of group G is the $(2\,m)$th power of a real transformation of this group for any even exponent $2\,m$.*

If the transformation T is defined by the system of equations

$$x'_r = a_{r1}\, x_1 + a_{r2}\, x_2 + \ldots . + a_{r,\,2n}\, x_{2n} \; (r = 1, 2, \ldots .\, \overline{2\,n - 1}, 2\,n),$$

let T_λ denote the transformation defined by the equations

$$x'_r = (a_{r1} x_1 + a_{r2} x_2 + \ldots . + a_{r,\,2n} x_{2n}) - \lambda\, x_r \; (r = 1, 2, \ldots\, \overline{2\,n - 1}, 2\,n),$$

λ being a root of multiplicity m of the characteristic equation of T.

* For an odd exponent $2\,m + 1$, any real transformation of group G is the $(2\,m + 1)$th power of a real transformation of this group.

The nullity * of T_λ is then at least one, and the nullity of successive powers of T_λ increases until a power of exponent $\mu \leqq m$ is attained whose nullity is equal to m. The nullity of the $(\mu + 1)$th and higher powers of T_λ is then also m. If we designate respectively by

$$m_1, m_2, \ldots . m_{\mu-1}, m_\mu = m,$$

the nullities of

$$T_\lambda, T^2_\lambda, \ldots . T^{\mu-1}_\lambda, T^\mu_\lambda,$$

then

$$m_1 \geqq m_2 - m_1 \geqq \ldots . \geqq m_\mu - m_{\mu-1} \geqq 1.$$

The numbers μ_1, μ_2, etc., may be termed the numbers *belonging* to the root λ of the characteristic equation of T.

If now T is the second power of a real transformation of group G, the numbers belonging to each negative root of the characteristic equation of T are all even. These conditions are probably not only necessary but sufficient in order that a real transformation T of group G may be the second power of a real transformation of this group.

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* The *nullity* of the transformation defined by the system of equations

$$x'_r = a_{r1} x_1 + a_{r2} x_2 + \ldots + a_{r,N} x_N \quad r = (1, 2, \ldots N),$$

is m if all the $(m-1)$th minors (the minor determinants of order $N-m+1$) are zero, but not all the mth minors (the minor determinants of order $N-m$) of the matrix.

$$\begin{matrix} a_{11}, a_{12}, \ldots \\ a_{21}, a_{22}, \ldots \\ \ldots\ldots\ldots \end{matrix}$$